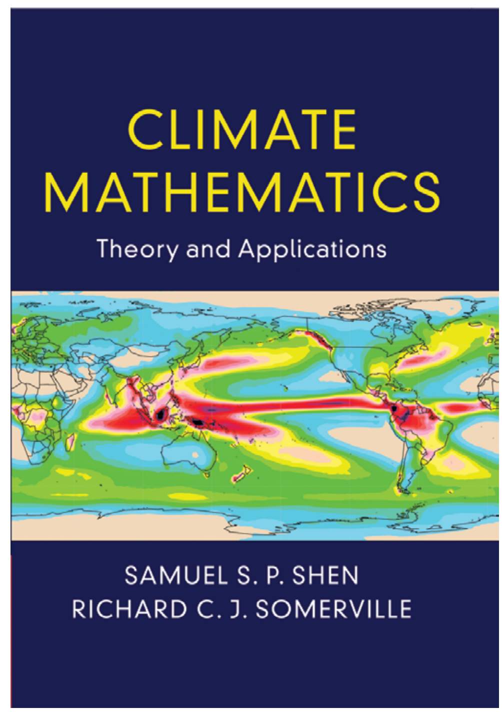
The Changing Math of Atmospheric Sciences

BAMS talked to Samuel S. P. Shen and Richard C. J. Somerville about their new book, _Climate Mathematics: Theory and Applications_ (Cambridge University Press, 2019).

Why do this book?
Undergrad students majoring in meteorology, oceanography, and climate science often take about six courses on differential and integral calculus, differential equations, linear algebra, basic statistics, and computer programming. These courses teach mathematics as a series of isolated and unrelated subjects. The curriculum has little relevance to a major such as climate science, and the pace of learning mathematics is too slow to meet the needs of the courses in the major subjects. We posed the following question to ourselves: how should we address the challenges and shortcomings of this current disconnected and time-consuming approach for climate science students to learn mathematics?



How do these shortcomings affect students?
The methodology not only makes these mathematical topics disconnected from one another, but also makes them even further disconnected from meteorology, oceanography, and climate science. Experience shows that students have difficulty in understanding why mathematics is taught in this way. They are unable to see how mathematics is used in solving "real-world" problems in their major fields. As a result, their professors may complain about the students' weak mathematics backgrounds. The students in turn are frustrated with the inappropriate and abstract mathematical knowledge that they have acquired from their mathematics, statistics, and computer science professors, who are unfamiliar with the applications of mathematics in meteorology, oceanography, and climate science.

How, then, did you address these concerns?
A course using this new textbook, *Climate Mathematics: Theory and Applications*, can replace the six disconnected and time-consuming courses with a single unified course, covering the mathematical topics that are most important and useful for climate science education and research.

How did you manage to combine six courses into one?
Knowing that the book would be limited to about 400 pages, we spent a lot of time deciding which topics to include and which topics to leave out. Our criteria were not mathematical ones such as beauty, elegance, and generality. Instead, we chose mathematical topics that were known to be useful in meteorology, oceanography, and climate science. For this reason, we included some important topics that are not taught in the usual mathematics courses, such as the singular value decomposition method to compute empirical orthogonal functions from a rectangular space–time data matrix.

Climate Mathematics is a unified course that can be taught in 1–3 semesters, depending on the students and their backgrounds. It provides the mathematics, statistics, and programming needed to prepare for coursework and research in climate science, meteorology, and oceanography. Overall, it presents selected concepts and techniques in linear algebra, statistics, calculus, and differential equations. All are discussed with climate science examples.

Who is the book for?
The book assumes no background beyond high school mathematics, and can support courses at different levels. *Climate Mathematics* is a highly flexible text for both undergraduate and graduate students. It provides climate science students with a mathematical background sufficient to prepare them to take not only upper-division undergraduate courses, but also most graduate-level climate science courses. *Climate Mathematics* can also serve as a reference for researchers and other professional climate scientists who need to refresh or modernize their quantitative skills.


Samuel Shen

What areas of math do you think people already in the field, educated earlier, are particularly lacking?
People educated in the traditional series of math courses often are unfamiliar with particular topics that are critical in climate science, and they also may not fully appreciate the value of recent advances in scientific computing. The singular value decomposition method, for example, has been absolutely central to important recent advances in our understanding of the El Niño/Southern Oscillation phenomenon, yet it is not part of the traditional curriculum.

What areas of math have they been learning that they don't tend to use or benefit from?
We have omitted many of the less useful topics found in the traditional courses. These include limits in Calculus I, infinite series and integration techniques in Calculus II, determinants in Linear Algebra, and series solutions in Differential Equations. These omitted materials are simply not used in typical current courses in the major subject areas of these students.

What are some of the ways the book modernizes skills?
Climate Mathematics strongly relies on recent key advances in scientific computing. Computational techniques are provided to visualize, analyze, and apply climate data. R code is featured in the book. R and equivalent Python codes are both available on the book's website. Exercises are provided in each chapter of *Climate Mathematics*. Our online supplements include datasets, images, and animations.

The book and its supplements enable students to analyze and visualize typical climate datasets, empowering them to carry out research and providing them with valuable skills. *Climate Mathematics* is a modern toolkit of useful mathematics, statistics, and computing for climate science in the era of big data.

How did you prepare to write such a comprehensive book?
We educated ourselves, especially in areas of one another's expertise. One of us (Samuel) is a mathematician and statistician who has worked collaboratively with several atmospheric and oceanographic and climate scientists. The other (Richard) is a dynamical meteorologist who has carried out extensive research in mathematical modeling of the atmosphere and the climate system. We taught one another during the three years we worked together on the book. Samuel also taught much

of the material in several courses before we began the book, and the lecture notes and student reactions from these courses were essential to writing the book.

We also carried out several independent research projects and reported on them in the book. For example, energy balance climate models omit explicit consideration of the motions of the atmosphere and the ocean. Thus, they are very simplified and idealized models of the Earth's climate. However, the moon has no atmosphere, no ocean, and no water. We developed an original energy balance model of the moon's climate and showed that it produces surface temperature distributions that compare well with lunar satellite observations.

What were some of the topics you had to relearn or revisit?

We found that some recent advances in science and technology led us to update some topics. These included the GPS-based radiosonde, a new hypsometric equation derived without the usual isothermal assumption, and a GPS-based planimeter as a smartphone app to measure the area of a region on the Earth's surface based on Green's theorem. We also elected to describe calculus using the simple Descartes direct approach without limits. This approach differs from that of a conventional calculus textbook. We tell the reader that: "You may find our treatment of calculus somewhat different from the way you were first taught this subject. Re-learning calculus may thus resemble returning to a place you have visited before and seeing it from a new perspective." We did not take this less usual approach just to be different, and we explain our reasons for choosing it. We quote Dr. Ferdinand Porsche, the brilliant automotive engineer, who said: "Change is easy. Improvement is far more difficult."

What surprised you?

Although we had both written books before, I think we were surprised by all the work we had to do in addition to writing the book itself. We created an extensive website (www.climatemathematics.org). We extensively tested all the computer codes. We wrote a 100-page solutions manual (available on a password-protected website) which provided answers, hints, and derivations for all the exercises in the book. The solutions manual is available from the publisher to qualified instructors on a password-protected website. It was also a very pleasant surprise to discover that after working closely together for three years on the book, we coauthors are still good friends!

Richard Somerville

Where do you go from here?

We hope that our book overcomes the curriculum shortcomings we mentioned, but there is another problem, which is that some areas of research in climate science require knowledge of additional specialized aspects of mathematics, and these are not included in this book. For example, numerical methods for ordinary and partial differential equations are essential for climate modeling, but they are beyond the scope of this book. We hope to include them in a future book on *Advanced Climate Mathematics*.